THE PROOF OF LEMMA 3.8.2

Lemma 8.3.2. Let A be a set, and let R be an EQUIVALENCE relation on A. [Comm! does not need def'n in the Proof]

For all $a, b \in A$, if aRb, then $[a] = [b]$.

Proof: Let $a, b \in A$ be given.

Suppose aRb. [NTS: $[a] \subseteq [b]$ AND $[b] \subseteq [a]$]

[Showing $[a] \subseteq [b]$]

Let $x \in [a]$ be given. [NTS: $x \in [b]$]

$\therefore$ xRa by def'n of $[a]$.

Since aRb and R is transitive, xRb.

$\therefore$ $x \in [b]$ by def'n of $[b]$.

$\therefore$ $[a] \subseteq [b]$, by Direct Proof.

[Showing $[b] \subseteq [a]$]

Let $y \in [b]$ be given. [NTS: $y \in [a]$]

$\therefore$ yRb by def'n of $[b]$.

$\therefore$ Since aRb and R is symmetric, bRa.

$\therefore$ Since R is transitive and yRb and bRa, yRa.

$\therefore$ $y \in [a]$, by def'n of $[a]$.

$\therefore$ $[b] \subseteq [a]$ by Direct proof.

$\therefore$ $[a] = [b]$ by definition of set equality.

$\therefore$ For all $a, b \in A$, if aRb, then $[a] = [b]$, by Direct proof. QED